TEMPORAL ASPECTS OF OTOACOUSTIC EMISSIONS

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The study of temporal aspects of otoacoustic emissions (OAEs) provides useful information concerning the sources of the emissions and helps to test and constrain cochlear emission models. A unified model, which successfully describes many of the characteristics of the various types of spontaneous and evoked otoaocoustic emissions and the related microstructure of the hearing threshold, has been recently formulated [1]. The model is based upon wave reflections via distributed spatial inhomogeneities and tall and broad cochlear activity patterns, as suggested by Zweig and Shera [2]. The model may be used to explore a number of issues relating to the temporal properties of OAEs. For example, in the case of the $2f_1 - f_2$ distortion product OAE (DPOAE), in which one of the primaries is on continuously and the other is pulsed on and off, the data can be interpreted in terms of approximate analytic time-domain solutions, which clearly exhibit the various characteristic latencies [3] as well as the level dependencies of cochlear wave reflections. It is also shown that when the higher frequency (f_2) primary is pulsed on, the latency of the earliest DPOAE component is significantly different from the f_2-sweep group delay which is inferred from DPOAE phase data for steady state primaries [*e.g.*, Bowman *et al.* [4]]. The model also allows an investigation of the so-called "filter build-up time" contribution [4] to the f_2-sweep group delay. It is shown that this contribution is small, and that the substantial difference between the f_2-sweep and f_1-sweep group delays, which has been attributed to the cochlear filter, may be largely accounted for by the implications of the approximate scale invariance of cochlear mechanics.

1 Introduction

The investigation of temporal aspects of otoacoustic emissions (OAEs) supplements the study of steady state properties, and thereby provides additional information relating the sources of these emissions. This information may be used to test and constrain the forms of interpretive cochlear emission models.

The most extensive previous studies of temporal aspects of OAEs have been those of the echolike responses to click, tone burst, or chirp stimuli (transient-evoked OAEs or TEOAEs) [5]. The temporal aspects of the suppression and release from suppression of spontaneous otoacoustic emissions (SOAEs) by pulsed external tones have also been investigated in the context of a simple limit-cycle-oscillator model of SOAEs [6]. Finally, there have been several studies of the temporal aspects of distortion product otoacoustic emissions produced in response to primary tones which are pulsed on and off [3,7].

A class of active-nonlinear cochlear models, which account for most of the characteristic variations with frequency of human OAEs and hearing threshold microstructure, has recently been developed and studied via numerical simulations and quasi-analytic approximate expressions [1]. The models are based upon cochlear wave reflections via distributed spatial inhomogeneities and tall/broad cochlear activity patterns, as suggested by Zweig and Shera [2]. For steady state properties of OAEs, the quasi-analytic expressions are expressible in terms of the cochlear wave basis states, *i.e.*, apical and basal moving waves in the absence of cochlear inhomogeneities and nonlinearity, and apical and basal cochlear wave reflectances, Ra and Rb. A preliminary application of the model to the case of a $2f_1 - f_2$ DPOAE from a steady state primary tone of frequency f_1 and a pulsed primary tone of frequency f_2 ($> f_1$) off [3] was successful in describing a number of characteristics of the temporal DPOAE response in relationship to the corresponding steady state response. In this paper, further details concerning the temporal characteristics of OAEs are given.

2 Synchronous Evoked Otoacoustics Emissions (SEOAEs)

2.1 Steady-State Response

For a steady state "calibrated" (complex) driving pressure $P_{dr}(t)$, given by

$$P_{dr}(t) = P_{dr}e^{i\omega t}, \tag{1}$$

the complex SEOAE amplitude is given by [1]

$$P_s(\omega) = P_{dr} - P_{me}(\omega)\frac{1 + R_a(\omega)\mathcal{R}_s(\omega)}{1 - R_a(\omega)R_b(\omega)}, \tag{2}$$

where $P_{dr} - P_{me}(\omega)$ is the ear canal pressure in the absence of apical cochlear wave reflections. R_a (R_b) is the the apical (basal) traveling wave reflectance which describes cochlear traveling wave ratios at the cochlear base in the presense (absence) of an acoustic driver of frequency ω in the ear canal. $\mathcal{R}_s(\omega) \cong e^{i\varphi_s(\omega)}$ is the ratio of derivatives of the cochlear wave basis functions at the base [1]. The characteristic fine structure of $P_s(\omega)$, however, stems mainly from the phase variation of R_a with ω [1,2,3].

2.2 Temporal Response to a Pulsed Primary

The stimulus for the case when P_{dr} is turned on at t=0, can be written as

$$P_{dr}^{on}(t) = P_{dr}e^{i\omega t}\Theta(t) = P_{dr}\int_{-\infty}^{+\infty} \frac{e^{i\omega' t}d\omega'}{\omega' - \omega - i\varepsilon}, \qquad \varepsilon \to 0^+, \tag{3}$$

where $\Theta(t) = 1$ if $t > 0$, and otherwise 0. The SEOAE signal is then

$$P_s^{on}(t;\omega) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{e^{i\omega' t}d\omega'}{\omega' - \omega - i\varepsilon} P_s(\omega). \tag{4}$$

The functional dependence of the integrand in Eq. (4) on ω' is very complex and makes the evaluation of $P_s(t;\omega)$ very challenging. As a result, the approximations which are necessary to permit the analytic evaluation of Eq. (4) will be left for a future publication, and only the main results of this analysis will be summarized here. We find for $t \geq 0$,

$$P_s^{on}(t;\omega) = P_{dr}e^{i\omega t} - P_{me}(\omega)e^{i\omega t} \times \left\{ 1 + \sum_{n=1}^{\infty} R_a^n(\omega)R_b^n(\omega)\mathcal{F}_{n-1}\left(\frac{\gamma_0}{2}t_n\right) \right.$$
$$\left. + \mathcal{R}_s(\omega)\sum_{n=1}^{\infty} R_a^n(\omega)R_b^{n-1}(\omega)\mathcal{F}_{n-1}\left[\frac{\gamma_0}{2}(t_n+\tau_b)\right] \right\}, \tag{5}$$

$$\mathcal{F}_n(\xi) = \Theta(\xi)\left[1 - e^{-\xi}e_n(\xi)\right], \qquad e_n(\xi) = \sum_{k=0}^{n}\frac{\xi^k}{k!}, \tag{6}$$

where γ_0 parametrizes the damping at position x, $t_n = t - n[2\tau(\hat{x}(\omega),\omega) + \tau_b]$, $k(x,\omega)$ is the (complex) traveling wavenumber, $\tau(x,\omega) \equiv \int_0^x dx' \mathrm{Re}[\partial k(x',\omega)/\partial\omega]$ is the travel time of a pressure wave of frequency ω from the base to the position x, $\tau_b \equiv -\partial \arg(R_b)/\partial\omega$, and $\hat{x}(\omega)$ is the tonotopic place for frequency ω. Finally, the expression for the turn-off of the f primary at $t = 0$, after the primary has been on steady state, is simply

$$P_s^{off}(t;\omega) = P_s(\omega) - P_s^{on}(t;\omega). \tag{7}$$

For $|R_aR_b| \ll 1$, the group delay of the steady state SEOAE, $P_s - P_{dr} - P_{me}$, can be shown to be $2\tau(\hat{x}(\omega),\omega)$, *i.e.*, the shortest latency of the temporal response.

3 Distortion Product Otoacoustic Emissions

3.1 Steady-State Response

For a steady state "calibrated" (complex) driving pressure $P_{dr}(t)$, given by

$$P_{dr}(t) = P_{1dr}e^{i\omega_1 t} + P_{2dr}e^{i\omega_2 t}, \tag{8}$$

the complex $2f_1 - f_2$ DPOAE amplitude is given by [1]

$$P_{dp}(\omega_1, \omega_2, \omega_{dp}) = \frac{P_\ell(\omega_1, \omega_2, \omega_{dp}) + R_a(\omega_{dp}) P_r(\omega_1, \omega_2, \omega_{dp})}{1 - R_a(\omega_{dp}) R_b(\omega_{dp})}. \quad (9)$$

Here P_ℓ is the (complex) amplitude of the overlap-region contribution to the ear canal pressure and the product $R_a P_r$ is the amplitude of the contribution arising from reflection of the distortion product (DP) energy at the DPOAE tonotopic site. Equation (9) characterizes, in a mathematical form, the "two-source" model of DPOAE generation, together with the effects of multiple internal reflection (which are described by the $1/(1 - R_a R_b)$ factor of Eq. (9)).

3.2 Temporal Response to a Pulsed f_2 Primary

As is discussed in Talmadge *et al.* [3], when considering the temporal response of DPOAEs, it is beneficial from both an experimental as well as theoretical basis to pulse only the f_2 primary. In particular, the latency of the $2f_1 - f_2$ DPOAE response to a change in the f_2 signal (*e.g.*, a shift in the f_2 level) is significantly longer than the latency of the response to a change in the f_1 signal. Because of this, the temporal characteristics of the DPOAE response are much more easily measured. The theoretical interpretation of the response of the DPOAE is simplified if only the f_2 (or f_1) primary is pulsed, rather than having both primaries pulsed simultaneously (see *e.g.*, Ref. 7).

As before, the stimulus condition in which the f_1 primary is on steady state, while the f_2 primary is turned on at $t = 0$, can be written as

$$P_{dr}^{on}(t) = P_{1dr} e^{i\omega_1 t} + P_{2dr} \int_{-\infty}^{+\infty} \frac{e^{i\omega_2' t} d\omega_2'}{\omega_2' - \omega_2 - i\varepsilon}. \quad (10)$$

The DPOAE signal for these conditions becomes in turn

$$P_{dp}^{on}(t; \omega_1, \omega_2, \omega_{dp}) = -\frac{e^{2i\omega_1 t}}{2\pi i} \int_{-\infty}^{+\infty} \frac{e^{-i\omega_2' t} d\omega_2'}{\omega_2' - \omega_2 + i\varepsilon} P_{dp}(\omega_1, \omega_2', 2\omega_1 - \omega_2'). \quad (11)$$

The approximations necessary to permit the analytic evaluation of this integration will again be left for a future publication. We find

$$P_{dp}^{on}(t; \omega_1, \omega_2, \omega_{dp}) \cong e^{+i\omega_{dp} t} P_\ell(\omega_1, \omega_2, \omega_{dp}) \sum_{n=0}^{\infty} R_a^n(\omega_{dp}) R_b^n(\omega_{dp}) \mathcal{F}_{2n}\left(\frac{\gamma_0 \tilde{t}_{nl,n}}{2}\right)$$

$$+ e^{+i\omega_{dp} t} R_a(\omega_{dp}) P_r(\omega_1, \omega_2, \omega_{dp}) \sum_{n=0}^{\infty} R_a^n(\omega_{dp}) R_b^n(\omega_{dp}) \mathcal{F}_{2n+2}\left(\frac{\gamma_0 \tilde{t}_{dp,n}}{2}\right), \quad (12)$$

$$\tilde{t}_{nl,n} = t - \hat{\tau}_2 - \tau(\hat{x}_2, \omega_{dp}) - n(2\hat{\tau}_{dp} + \tau_b), \quad (13)$$

$$\tilde{t}_{dp,n} = t - \hat{\tau}_2 - 2\hat{\tau}_{dp} + \tau(\hat{x}_2, \omega_{dp}) - n(2\hat{\tau}_{dp} + \tau_b), \quad (14)$$

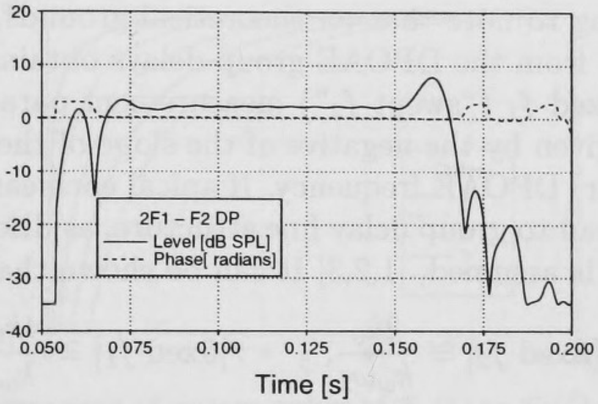

Figure 1: Typical model calculation showing interference notches in the transient DPOAE response, as describe by Eqs. (12)-(15).

where $\hat{\tau}_2 = \tau(\hat{x}_2, \omega_2)$ $\hat{\tau}_{dp} = \tau(\hat{x}_{dp}), \omega_{dp})$, $\hat{x}_2 = \hat{x}(\omega_2)$ and $\hat{x}_{dp} = \hat{x}(\omega_{dp})$.

The expression for P_{dp} after the turn-off of the f_2 primary at $t = 0$, after both primaries have been on steady-state, is

$$P_{dp}^{off}(t; \omega_1, \omega_2, \omega_{dp}) = P_{dp}(\omega_1, \omega_2, \omega_{dp}) - P_{dp}^{on}(t; \omega_1, \omega_2, \omega_{dp}). \qquad (15)$$

4 Discussion

The plot of $P_{dp}^{on,off}$ shown in Fig. 1 displays a number of typical features of Eq. (12). Destructive interference notches may appear shortly after the turn on or off of the f_2 primary, depending on whether $|R_a P_r / P_\ell|$ is greater or less than 1. The train of interference nulls after the f_2 primary is turned off will occur if $\arg[R_a R_b] = (2n+1)\pi (n = \text{integer})$. This simulation suggests that the study of the temporal aspects of DPOAEs may be useful for extracting the clinically interesting generator component, even when there are significant apical and basal cochlear wave reflections which complicate the interpretation of steady state DPOAE data [1,3]. The shortest latency of the temporal response is $\tau_{nl} = \hat{\tau}_2 + \tau(\hat{x}_2, \omega_{dp})$, which corresponds to the forward travel time of the f_2 primary cochlear wave from the base ($x = 0$) to the DP generation region (near $\hat{x}_2$), plus the reverse travel time of the initially generated DP wave from $\hat{x}_2$ to the cochlear base. Using the assumption of scale invariance of cochlear mechanics [1,3], it can be shown that

$$\tau_{nl} \cong (\hat{k} + k_0)/k_\omega \omega_2, \qquad (16)$$

where $\hat{k} = \text{Re}[k(\hat{x}(\omega), \omega]$, k_0 is a constant related to the geometry of the cochlea at the base, and k_ω is the exponential constant in the basilar membrane place-frequency map, $\omega(x) = \omega_{base} e^{-k_\omega \omega}$.

It is interesting to note that on theoretical grounds, this latency will be generally different from the DPOAE group delays obtained using the fixed f_2 ("swept f_1") or fixed f_1 ("swept f_2") measurement paradigms. The DPOAE group delays are given by the negative of the slope of the DPOAE phase with respect to (angular) DPOAE frequency. If apical cochlear wave reflections are neglected (these lead to group delay fine structure, as discussed in Ref. 3), and if scale invariance is assumed, [1,2,3] It can be shown that

$$\tau[\text{fixed } f_2] \cong \frac{2k_0}{k_\omega \omega_2}, \qquad \tau[\text{fixed } f_1] \cong \frac{4k_0\omega_1}{k_\omega \omega_2^2}. \tag{17}$$

The difference between these two group delays found in typical steady-state DPOAE data [4] can thus be understood theoretically without having to invoke the concept of the "filter buildup time" contribution to $\tau[\text{fixed } f_1]$. This latter effect can in fact be shown to be quite small. (The details of this analysis will be left for a future publication). Using the assumption of scale invariance, it is also possible to evaluate $\hat{\tau}_2$ directly [1,2],

$$\hat{\tau}_2 = \int_0^{\hat{x}_2} dx' \frac{\partial k(x', \omega_2)}{\partial \omega} = \frac{1}{k_\omega \omega_2} \int_0^{\hat{x}_2} dx' \frac{\partial k(x', \omega_2)}{\partial x'} \cong \frac{\hat{k}}{k_\omega \omega_2}. \tag{18}$$

Also, from fixed f_2 measurements [1], $\hat{k} \cong 3.6k_0$, which gives

$$\tau[\text{fixed } f_2] \approx 0.6\hat{\tau}_2, \qquad \tau[\text{fixed } f_1] \approx 1.1\frac{\omega_1}{\omega_2}\hat{\tau}_2; \qquad \tau_{nl} \cong 1.28\hat{\tau}_2. \tag{19}$$

Consequently, neither of the conventional DPOAE group delays correspond to τ_{nl}. An alternative measure of the steady state DPOAE, that directly relates to the travel time $\hat{\tau}_{dp}$, is based on the fine structure spacing criteria [1,2] $(2\hat{k}/k_\omega)(\Delta f/f) = 2\pi$, where Δf is the fine structure spacing at frequency f. Combining this criterion with Eq. (18) gives

$$\Delta f[f] \cong 1/2\hat{\tau}_{dp}, \tag{20}$$

which just relates the average DPOAE fine structure spacing to the travel time of the DP wave between $x = 0$ and $x = \hat{x}_{dp}$.

Finally, it is useful to compare the results of detailed calculations outlined in this paper to the ones that one might naively assume, such as Eqs. (21)–(27) of Ref. 3. Comparing these models, the main difference is the appereance of the $e_n(\xi)$ term in the exponential decay $1 - e^{-\xi}e_n(\xi)$, which is not present in the naive model. Figure 2 illustrates how this additional term influences the temporal characteristics of different components to the OAE response. Intepreting the more complex temporal behavior of the higher order components in terms of the naive $1 - e^{-\xi}$ model which they resemble would give results that are systematically too small for the damping and too large for the delay.

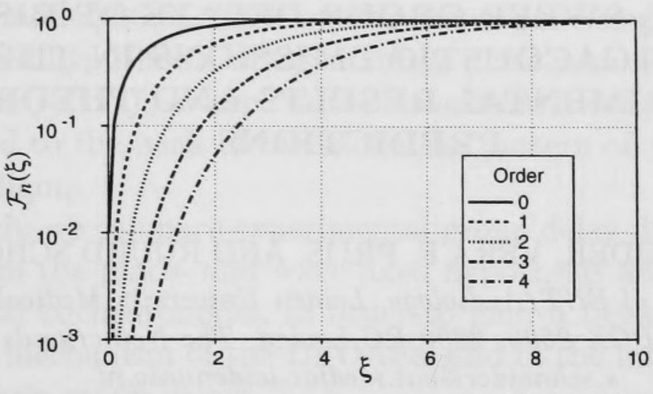

Figure 2: Comparison of various orders of $\mathcal{F}_n(\xi)$ to $\mathcal{F}_0(\xi) = 1 - e^{-\xi}$.

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References

1. Talmadge, C. L., Tubis, A., Piskorski, P., Long, G. and Piskorski, P. (1998). Modeling otoacoustic emission and hearing threshold fine structures. *J. Acoust. Soc. Am.* **130**, 1517–1543.
2. Zweig, G. and Shera, C. A. (1995). The origins of periodicity in the spectrum of evoked otoacoustic emissions. *J. Acoust. Soc. Am.* **98**, 2018–2047.
3. Talmadge, C. L., Long, G. R., Tubis, A. and Dhar, S. (1999). Experimental confirmation of the two-source model of distortion product otoacoustic emissions. *J. Acoust. Soc. Am.* **105**, 275–292.
4. Bowman, D. M., Eggermont, J. J., Brown, D. K. and Kimberly, B. P. (1998). Estimating cochlear filter response properties from distortion product otoacoustic emission (DPOAE) phase delay measurements in normal hearing human adults. *Hear. Res.* **119**, 14–26.
5. Robinette, M. S. and Glattke, T. J. (Eds.) (1997). *Otoacoustic Emissions: Clinical Applications*, (Thieme, New York).
6. Murphy, W. J., Talmadge, C. L., Tubis, A. and Long, G. R. (1996). Relaxation dynamics of spontaneous otoacoustic emissions perturbed by external tones: III. Response to a single tone at multiple suppression levels. *J. Acoust. Soc. Am.* **100**, 3979–3982.
7. Whitehead, M. L. Stagner, B. B. Martin, G. K. and Lonsbury-Martin, B. L. (1996). Visualization of the onset of distortion-product otoacoustic emissions, and measurement of their latency. *J. Acoust. Soc. Am.* **100**, 1663–1679.